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QUARTERLY PROGRESS REPORT
FOR APRIL, MAY, AND JUNE 1970
Under Contract N00024-70-C-1279

NAVAL SHIP SYSTEMS COMMAND
Contract N00024-70-C-1279
Proj. Ser. No. SF 11552-001, Task 8118

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MAR 8 1977

(11) 21 July 1970

(12) 41 p.

(6)

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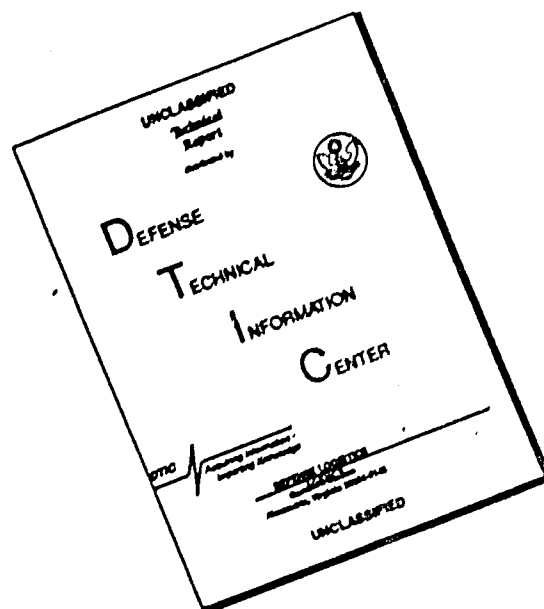AUSTIN, TEXAS 78712

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ABSTRACT

In Section I it is shown by using the simplest reflecting
surface--an infinite, pressure release plane surface--
that at least the Fresnel phase approximation and a reasonable
directivity pattern approximation are necessary before accurate
predictions in surface reflection and scattering problems can
be made. The average Rayleigh reflection coefficient for random
penetrable surfaces of various roughnesses is calculated in
Section II. In Section III an integral expression is derived
for the acoustic field of a point source in a surface duct with
a rough boundary. The integral was obtained by Green's functions
and includes the rough surface condition by means of an effective
impedance condition which also depends on the properties of the
medium.

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I. THE REFLECTION OF AN ACOUSTIC BEAM
BY AN INFINITE, PRESSURE RELEASE PLANE SURFACE

The prediction of the field scattered by a known surface configuration or, conversely, the prediction of the surface configuration from its scattered field is a difficult and complex problem. Due to the extreme complexity of the problem, all of the solutions of the integral equations obtained to date have contained numerous approximations. One of the fundamental difficulties associated with surface scattering has been the determination of the ranges of validity and the interaction of the various approximations.

The usual approximations can be put into five categories:
(1) formulation, (2) boundary values, (3) source directivity, (4) phase approximation, and (5) surface representation. It was shown in the Final Report under Contract N00024-69-C-1275 (April 1970) that the potential formulation of reflection and scattering problems has distinct advantages over the Helmholtz' and Green's functions formulations and, in the limit of a plane surface, reduces to the Rayleigh-Sommerfeld formula. The boundary value for a plane, pressure release surface is known exactly. The boundary values for arbitrary penetrable surfaces will be the subject of later progress reports. Surface representations for randomly rough surfaces will also be treated further in future reports.

At this time only an infinite, pressure release plane surface will be considered, since this is the simplest of surfaces and the results are easily verified by the method of images. By treating a plane surface, attention can be focused exclusively on the phase and

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source directivity approximations that will be necessary in the treatment of arbitrary penetrable surfaces and the inverse problem, and certain misconceptions about the phase and directivity approximations can be exposed.

To gain insight into what approximations might be necessary, it is useful to recall two experimentally established properties of the intensity reflected by a pressure release plane surface: (1) the range dependence is that predicted by geometrical acoustics, namely $1/(r_{oo}+r_{lo})$, and (2) the results predicted by the image technique are indeed obtained. Drawings AS-69-478 and AS-69-479 demonstrate these properties. In light of the experimental facts, two conclusions are inescapable: (1) the Fraunhofer phase approximation is <u>never</u> correct when one is concerned with surface reflection or scattering since it predicts a $1/r_{oo}r_{lo}$ spreading loss; and (2) the analytic solution should reduce to the image solution. The first conclusion applies to arbitrarily rough surfaces since in the limit as the roughness approaches zero, the results must reduce to a plane surface. Hence, second and possibly higher order terms must be used in the expansion of r_o and r_1 in the phase (Fresnel approximation); otherwise, the correct range dependence will never be obtained.

In previous reports (see, for example, Final Report under Contract N00024-70-C-1275), it has been shown that the pressure reflected from a plane pressure release surface to some receiver at A is given by:

$$p(A) = \frac{ik}{2\pi} \iint_{\Sigma'} P_o \frac{e^{ik(r_o+r_1)}}{r_o r_1} \hat{n} \cdot \hat{e}_1 dS \quad , \qquad (1)$$

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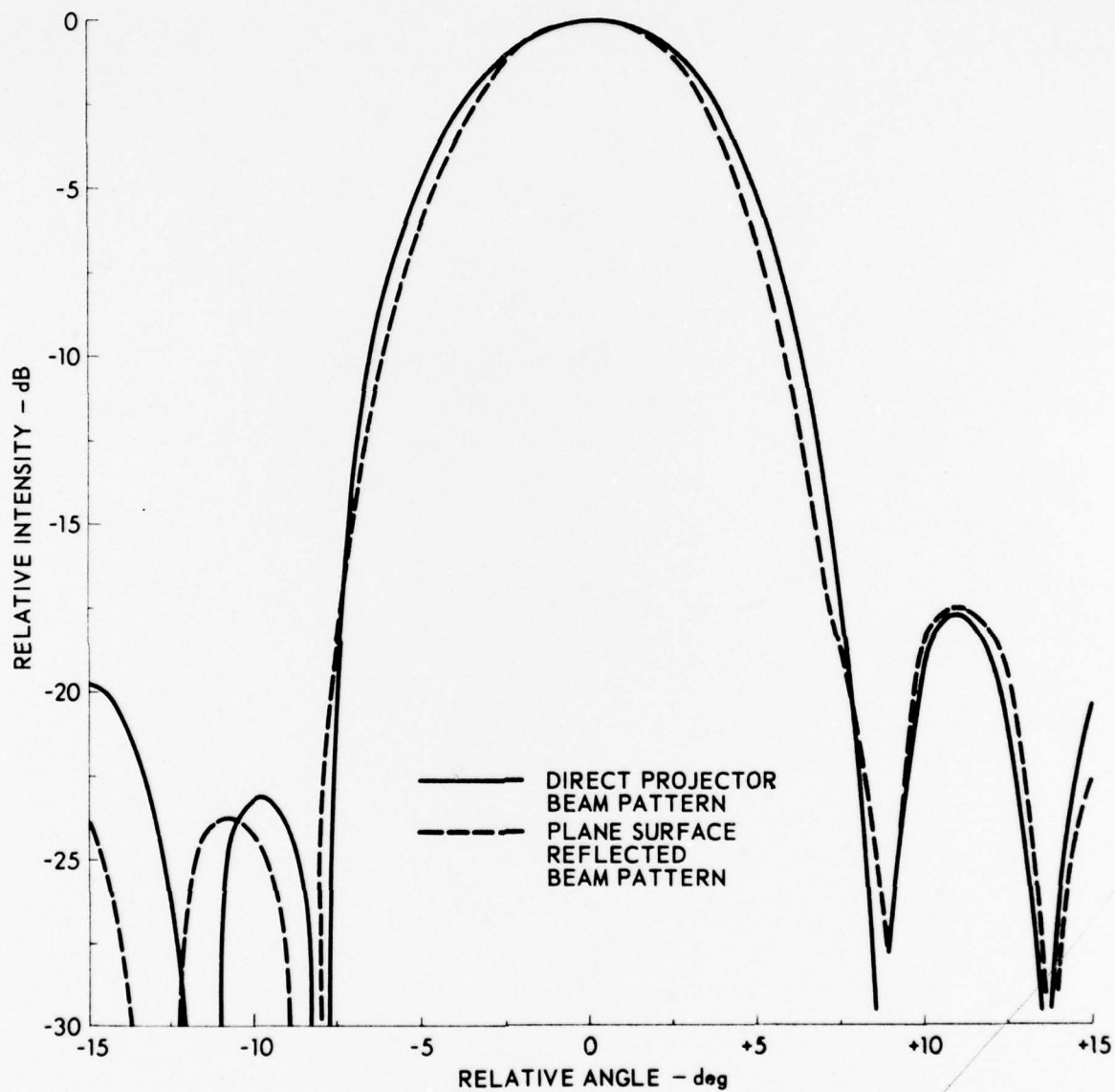
COMPARISON OF DIRECT AND PLANE SURFACE
REFLECTED PROJECTOR BEAM PATTERNS

3
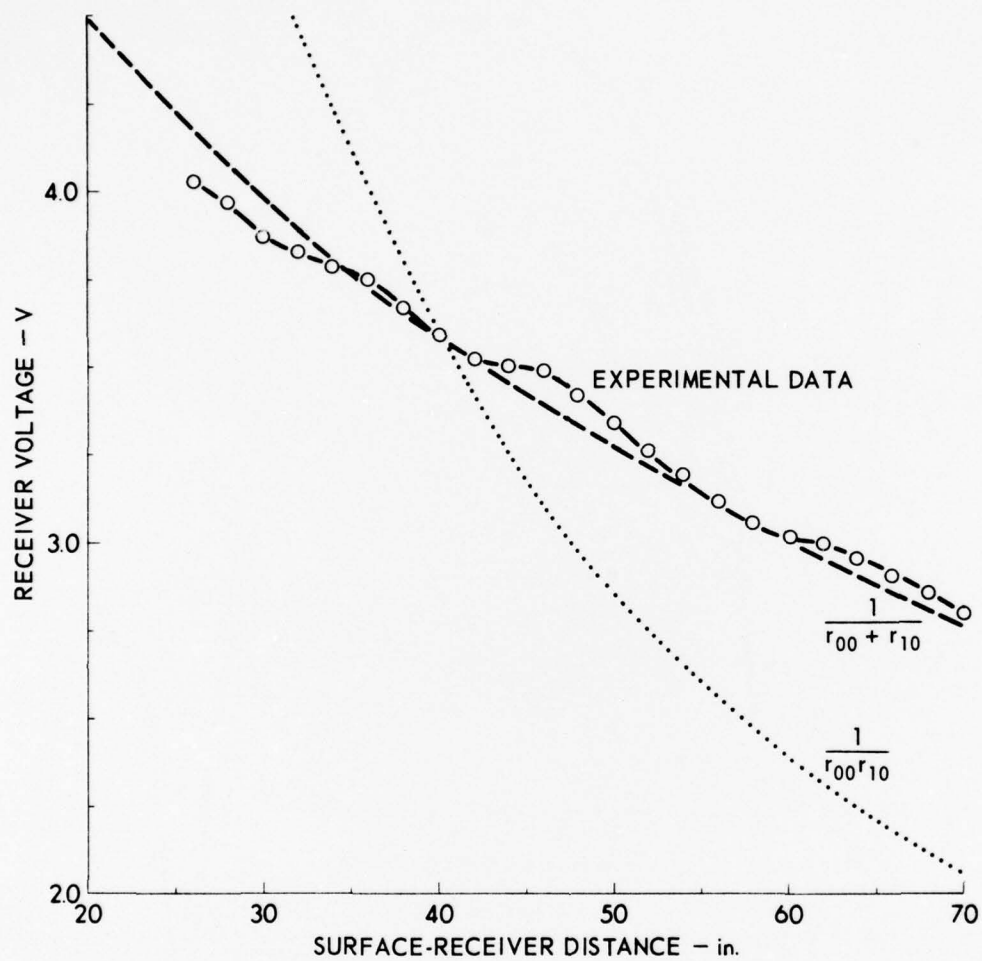
COMPARISON OF THE EXPERIMENTALLY MEASURED RANGE
DEPENDENCE OF THE PLANE SURFACE REFLECTED PRESSURE
FIELD AND RANGE DEPENDENCES OF $\dfrac{1}{r_{00}r_{10}}$ AND $\dfrac{1}{r_{00} + r_{10}}$
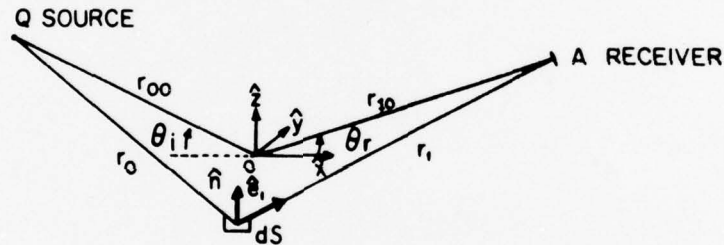
where

1) if the amplitude factor is suppressed, P_o is simply the directivity pattern of the source,

2) r_o and r_1 are the distances from the source and the receiver respectively to the surface element dS,

3) $\hat{n}$ is the unit normal out of the surface at the point enclosed by dS,

4) $\hat{e}_1$ is the unit vector pointing from the surface element dS along r_1 to the receiver, and

5) Σ' represents the insonified surface area.

The only assumption contained in this result is kr≫1. The geometry is depicted in the accompanying diagram.

The origin is fixed at the point where the axis of the acoustic beam intersects the plane. From simple trigonometry, it can be shown for a plane:

$$\hat{n} \cdot \hat{e}_1 = \frac{r_{1o} \sin\theta_r}{r_1} \quad , \tag{2}$$

and

$$dS = dxdy \quad . \tag{3}$$

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Note that r_{lo}, r_{oo}, θ_r, and θ_i are constants, while r_l and r_o are variables.

In general, Eq. (1) cannot be evaluated without making certain approximations. Previously, it was pointed out that at least the Fresnel approximation was required in the phase. The Fresnel phase approximation is given by the equation[1]

$$ik(r_l + r_o) = ik(r_{lo} + r_{oo} + ax + \frac{1}{R_1} x^2 + \frac{1}{R} y^2) + \text{higher order terms} \quad , \tag{4}$$

where

$$a = \cos\theta_i - \cos\theta_r \quad ,$$
$$R = 2r_{oo}r_{lo}/(r_{oo} + r_{lo}) \quad ,$$
$$R_1 = 2r_{oo}r_{lo}/(r_{oo}\sin^2\theta_r + r_{lo}\sin^2\theta_i) \quad .$$

Elsewhere r_o and r_l will be replaced by r_{oo} and r_{lo}.

All that remains now is to find a suitable approximation to the directivity pattern. For a nonisotropic source there are essentially two tractable approximations for the directivity pattern. The first is the simple aperture type function where it is assumed the insonification is a constant over the region of the surface defined by the -3 dB points of the source and zero elsewhere on the surface. The geometry is shown in Dwg. AS-69-635. If the beamwidth of the source is Φ deg, then the following expressions hold:

$$\Delta_1 = r_{oo}\sin(\theta/2)/\sin(\theta_i + \Phi/2) \quad , \tag{5}$$

$$\Delta_2 = r_{oo}\sin(\Phi/2)/\sin(\theta_i - \Phi/2) \quad , \tag{6}$$

$$\ell = r_{oo}\tan(\Phi/2) \quad , \text{ and} \tag{7}$$

$$y_o = \frac{\Delta_2 - \Delta_1}{2} \quad . \tag{8}$$

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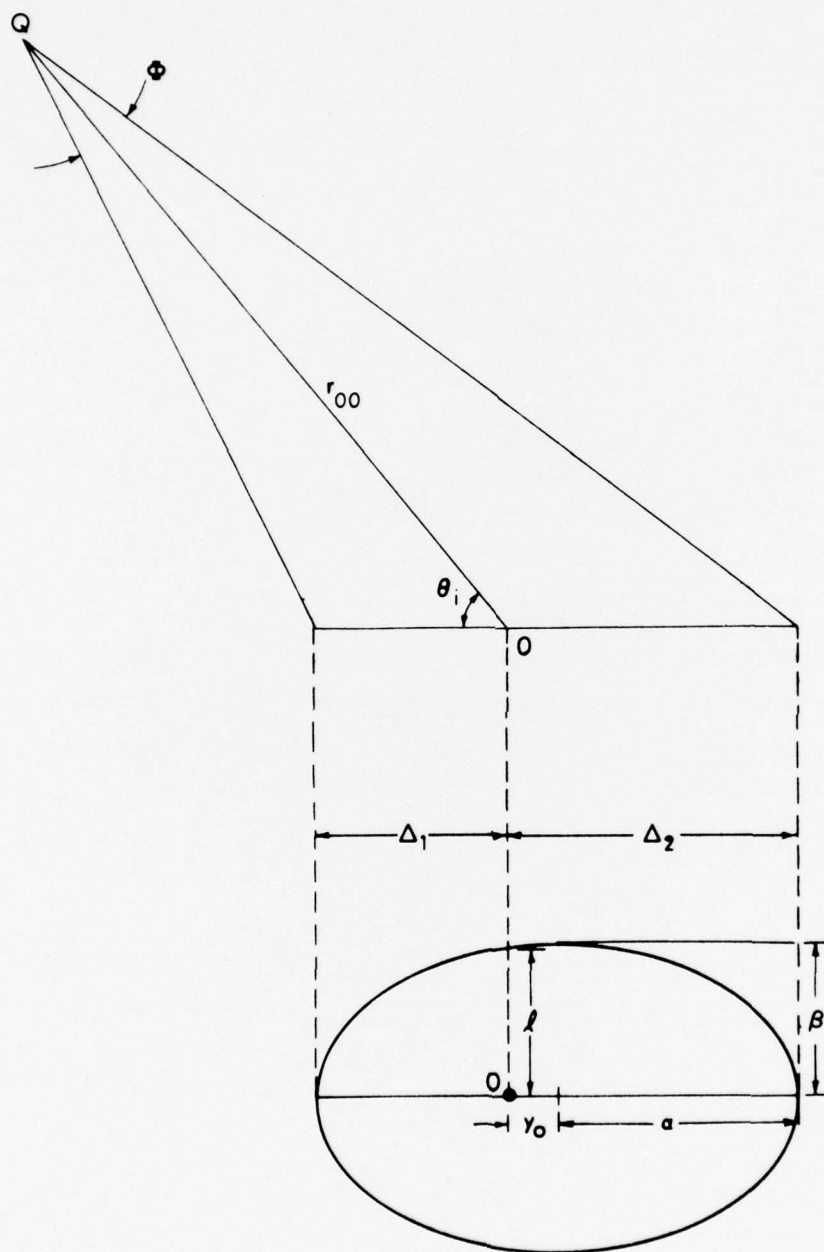
INSONIFIED AREA GEOMETRY

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Then:

$$K = 3/20 \, \log_{10} e \quad , \tag{9}$$

$$\alpha = \frac{\Delta_2 + \Delta_1}{2} \quad , \text{ and} \tag{10}$$

$$\beta = \frac{\ell \alpha}{\sqrt{y_o^2 + \alpha^2}} \tag{11}$$

where

1) K is a constant which provides the -3 dB value at the appropriate point off the beam axis,

2) α is the semimajor axis of the insonified ellipse, and

3) β is the semiminor axis.

Note that the effect of the range differences from one point on the ellipse to another has been ignored in setting the -3 dB points. These expressions are calculated in detail in the Quarterly Progress Report under Contract N00024-69-C-1275 for April - June 1969. In the calculations, a beamwidth, Φ, of 8 deg will be assumed since this is the beamwidth of the source used in the experimental program.

If now the ellipse is approximated by a rectangle of dimensions 2ℓ, $\Delta_2 + \Delta_1$, then the pressure reflected by a plane surface where the directivity pattern is assumed to be a simple aperture may be calculated.[1] Using Eqs. (2), (3), and (4) in Eq. (1) gives

$$p(A) = \frac{ik \, \sin\theta_r}{2\pi r_{oo} r_{lo}} \, e^{ik(r_{oo} + r_{lo})} \int_{-\Delta_1}^{\Delta_2} \int_{-\ell}^{\ell} e^{ik\left(ax + \frac{x^2}{R_1} + \frac{y^2}{R}\right)} dxdy \quad . \tag{12}$$

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The integrals in Eq. (12) are by definition Fresnel integrals and upon forming the product of p(A) and its complex conjugate, the intensity is obtained:

$$Ip = p^*(A)p(A) = \frac{\left[C_\ell\left(C_{\Delta_1} + C_{\Delta_2}\right) - S_\ell\left(S_{\Delta_1} + S_{\Delta_2}\right)\right]^2 + \left[S_\ell\left(C_{\Delta_1} + C_{\Delta_2}\right) + C_\ell\left(S_{\Delta_1} + S_{\Delta_2}\right)\right]^2}{\left(r_{oo} + r_{1o}\right)\left(r_{oo} + r_{1o}\sin^2\theta_i / \sin^2\theta_i\right)} \quad , \tag{13}$$

where

$$C(x) = \int_0^x \cos\left(\frac{\pi}{2} u^2\right) du \quad , \tag{14}$$

and

$$S(x) = \int_0^x \sin\left(\frac{\pi}{2} u^2\right) du \quad . \tag{15}$$

Then

$$C_\ell = C\left[\ell\sqrt{\frac{2k}{\pi R}}\right] \quad , \tag{16}$$

$$C_{\Delta_1} = C\left[\left(\Delta_1 - \frac{1}{2} aR_1\right)\sqrt{\frac{2k}{\pi R_1}}\right] \quad , \tag{17}$$

$$C_{\Delta_2} = C\left[\left(\Delta_2 + \frac{1}{2} aR_1\right)\sqrt{\frac{2k}{\pi R_1}}\right] \quad , \tag{18}$$

$$S_\ell = S\left[\ell\sqrt{\frac{2k}{\pi R}}\right] \quad , \tag{19}$$

$$S_{\Delta_1} = S\left[\left(\Delta_1 - \frac{1}{2} aR_1\right)\sqrt{\frac{2k}{\pi R_1}}\right] \quad , \text{ and} \tag{20}$$

$$S_{\Delta_2} = S\left[\left(\Delta_2 + \frac{1}{2} aR_1\right)\sqrt{\frac{2k}{\pi R_1}}\right] \quad . \tag{21}$$

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It is, however, possible to make a considerably better approximation to the directivity pattern. If the side lobes may be ignored, a suitable functional form is

$$P_o = exp\left[-K\psi^2/(\Phi/2)^2\right] \quad , \tag{22}$$

where

1) K and Φ have been previously defined, and
2) ψ is the angular displacement measured from the beam axis.

Since the integration is to be performed over the surface, it is necessary to express Eq. (22) as a function of the surface variables x and y. A useful, analytically tractable approximation to Eq. (22) is

$$P_o = exp\left[-K\left(\frac{x^2}{\alpha^2} + \frac{y^2}{\beta^2}\right)\right] \quad . \tag{23}$$

This form was suggested by Clay,[2] and Horton and Muir.[3] The directivity pattern given by Eq. (23) represents a single lobe* source whose surface insonification is an ellipse of semimajor axis α, and semiminor axis β. With the introduction of Eq. (23) the integration limits may now be taken as infinite for most practical cases. This consideration can be very important in connection with arbitrary surfaces, since often the integrability of the equations will depend on the extent of the limits.

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*It is possible to generalize Eq. (23) to a multi-lobe directivity pattern; for an N-lobe source the equation becomes

$$P_o = \sum_{i=1}^{N} e^{-K_i\left(\frac{x^2-\delta_i^2}{\alpha_i^2} + \frac{y^2-\epsilon_i^2}{\beta_i^2}\right)} \quad .$$

In Dwg. AS-70-800, the experimentally measured directivity pattern of the 100 kHz source used in scattering experiments is compared with Eq. (22). Drawing AS-70-809 compares Eqs. (22) and (23) for various source grazing angles

Now the intensity reflected from a plane, pressure release surface can be calculated. Substituting Eqs. (4) and (23) into Eq. (1) and taking the product of $p(A)$ with its complex conjugate gives

$$I_p = p^*(A)p(A) = \left(\frac{k\,\sin\theta_r}{2\pi r_{oo} r_{lo}}\right)^2 \iiiint_{-\infty}^{+\infty} e^{-K\left(\frac{x^2+x'^2}{\alpha^2} + \frac{y^2+y'^2}{\beta^2}\right)}$$

$$\cdot\, e^{\,i\left[ka(x-x') + \frac{k}{R_1}(x^2-x'^2) + \frac{k}{R}(y^2-y'^2)\right]} dx\,dy\,dx'\,dy' \quad .$$

(24)

Make the transformations $\delta = x-x'$, $\eta = y-y'$, $\bar{x} = (x+x')/2$, and $\bar{y} = (y+y')/2$ then the integrations follow easily, yielding

$$I_p = \frac{\exp\left[-KR_1^2 a^2/2\alpha^2(1+P)\right]}{(r_{oo}+r_{lo})(r_{oo} + r_{lo}\sin^2\theta_i/\sin^2\theta_r)\sqrt{(1+P)(1+Q)}} \quad ,$$

(25)

where

$$P = \left(KR_1/k\alpha^2\right)^2, \text{ and}$$

$$Q = \left(KR/k\beta^2\right)^2.$$

P and Q are approximately zero for the case of interest here. (This is merely a restatement of $kr \gg 1$.)

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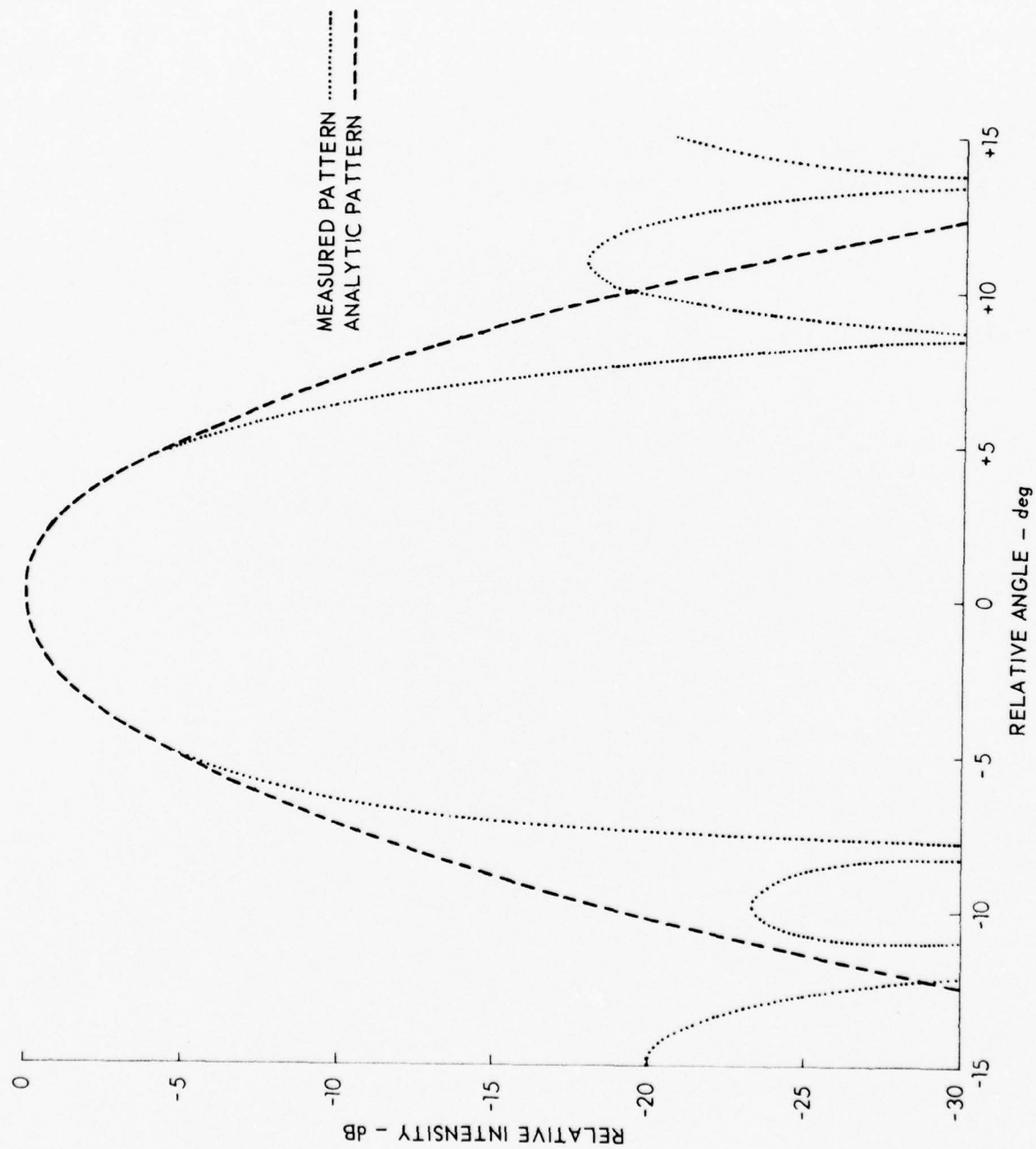
COMPARISON OF MEASURED DIRECTIVITY PATTERN
AND ANALYTIC DIRECTIVITY PATTERN

MEASURED PATTERN ....
ANALYTIC PATTERN ----------

RELATIVE ANGLE – deg

RELATIVE INTENSITY – dB

COMPARISON OF ANALYTIC
DIRECTIVITY PATTERN (Eq. 22)
AND THE APPROXIMATE
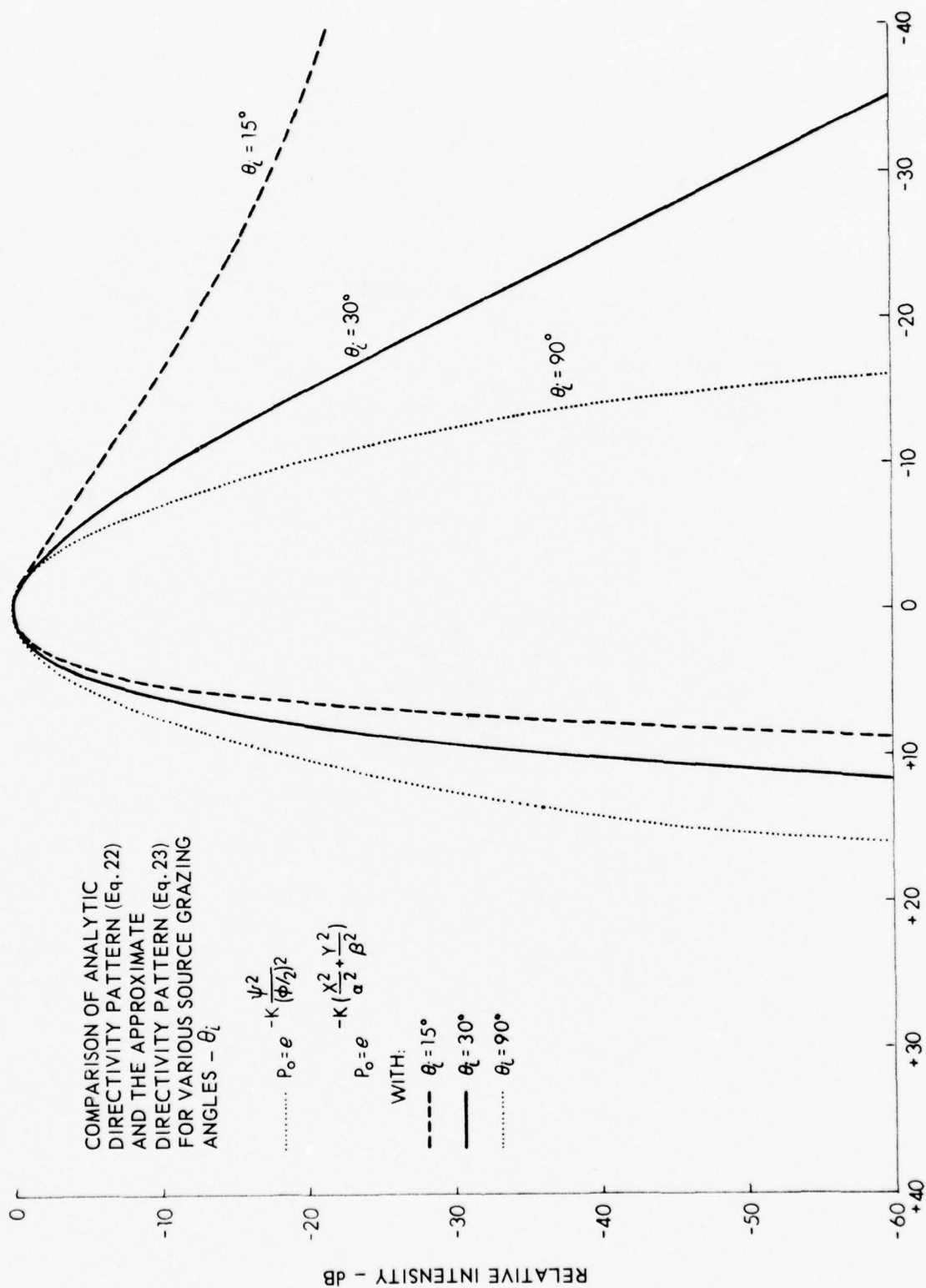DIRECTIVITY PATTERN (Eq. 23)
FOR VARIOUS SOURCE GRAZING
ANGLES — θ_i

$P_0 = e^{-K \frac{\psi^2}{(\phi/2)^2}}$

$P_0 = e^{-K(\frac{X^2}{\alpha^2} + \frac{Y^2}{\beta^2})}$

WITH:

   $\theta_i = 15°$

   $\theta_i = 30°$

   $\theta_i = 90°$

$\theta_i = 15°$

$\theta_i = 30°$

$\theta_i = 90°$

RELATIVE ANGLE FROM BEAM AXIS — deg

RELATIVE INTENSITY — dB

-40 -30 -20 -10 0 +10 +20 +30 +40

0 -10 -20 -30 -40 -50 -60

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The solution obtained using image techniques is

$$I_p^{image} = \frac{P_o^2}{(r_o'+r_1')^2} = \frac{exp\left[-2K\psi^2/(\Phi/2)^2\right]}{(r_o'+r_1')^2} \quad , \qquad (26)$$

where ψ is now the angle to
the receiver measured from the
beam axis of the image source.
The geometry is given in the
accompanying diagram. Through
simple geometry, ψ is found in
terms of r_{oo}, r_{1o}, θ_i, and θ_r:
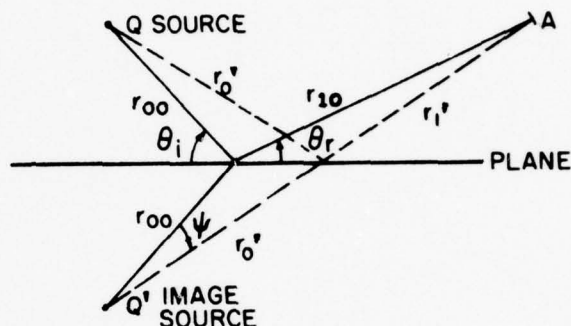
$$\psi = Arctan\left[\frac{r_{1o}\sin(\theta_i-\theta_r)}{r_{oo}+r_{1o}\cos(\theta_i-\theta_r)}\right] \quad , \qquad (27)$$

and by the Law of Cosines

$$(r_o'+r_1')^2 = r_{oo}^2+r_{1o}^2+2r_{oo}r_{1o}\cos(\theta_i-\theta_r) \quad . \qquad (28)$$

In the specular direction, examination of Eqs. (25) and (26) reveals

$$I_p = I_p^{image} = \frac{1}{(r_{oo}+r_{1o})^2} \quad . \qquad (29)$$

The image solution Eq. (26) is compared with Eqs. (13) and (25)
for various source grazing angles and receiver ranges in Dwgs. AS-70-801,
AS-70-802, and AS-70-803. The Fresnel solution with the realistic
directivity pattern (Modified Fresnel solution) is clearly superior to
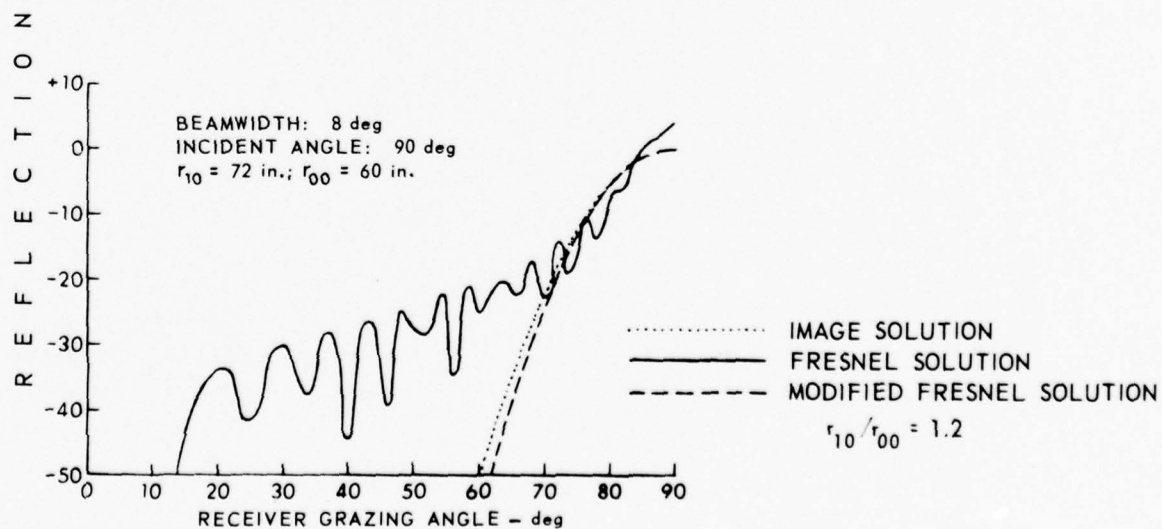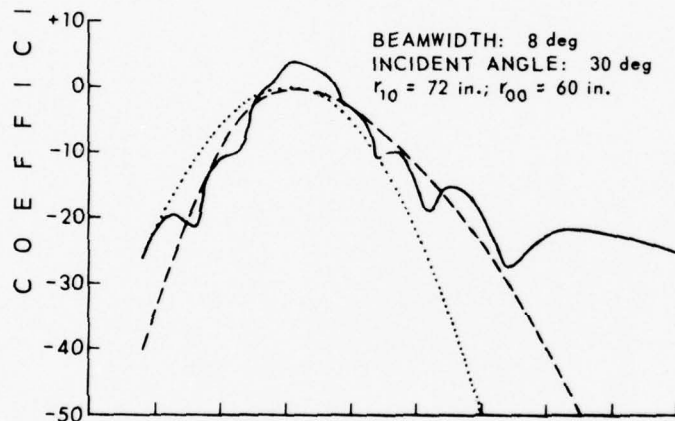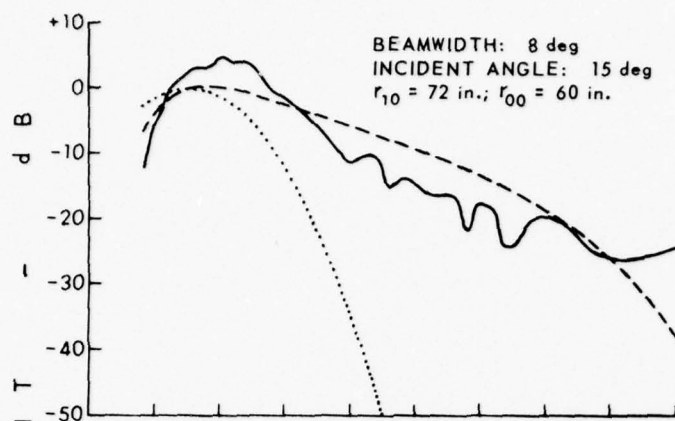
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COMPARISON OF THE IMAGE SOLUTION WITH THE FRESNEL
AND MODIFIED FRESNEL SOLUTION FOR A PLANE

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BEAMWIDTH: 8 deg
INCIDENT ANGLE: 15 deg
r_{10} = 200 in.; r_{00} = 60 in.

BEAMWIDTH: 8 deg
INCIDENT ANGLE: 30 deg
r_{10} = 200 in.; r_{00} = 60 in.

BEAMWIDTH: 8 deg
INCIDENT ANGLE: 90 deg
r_{10} = 200 in.; r_{00} = 60 in.

··········· IMAGE SOLUTION
————— FRESNEL SOLUTION
– – – – MODIFIED FRESNEL SOLUTION
r_{10}/r_{00} = 3.3

COMPARISON OF THE IMAGE SOLUTION WITH THE FRESNEL
AND MODIFIED FRESNEL SOLUTION FOR A PLANE

16

BEAMWIDTH: 8 deg
INCIDENT ANGLE: 15 deg
$r_{10} = 2000$ in.; $r_{00} = 60$ in.

BEAMWIDTH: 8 deg
INCIDENT ANGLE: 30 deg
$r_{10} = 2000$ in.; $r_{00} = 60$ in.

BEAMWIDTH: 8 deg
INCIDENT ANGLE: 90 deg
$r_{10} = 2000$ in.; $r_{00} = 60$ in.

.... IMAGE SOLUTION
———— FRESNEL SOLUTION
– – – – MODIFIED FRESNEL SOLUTION
$r_{10}/r_{00} = 33.3$

REFLECTION COEFFICIENT – dB

RECEIVER GRAZING ANGLE – deg

COMPARISON OF THE IMAGE SOLUTION WITH THE FRESNEL
AND MODIFIED FRESNEL SOLUTION FOR A PLANE

17
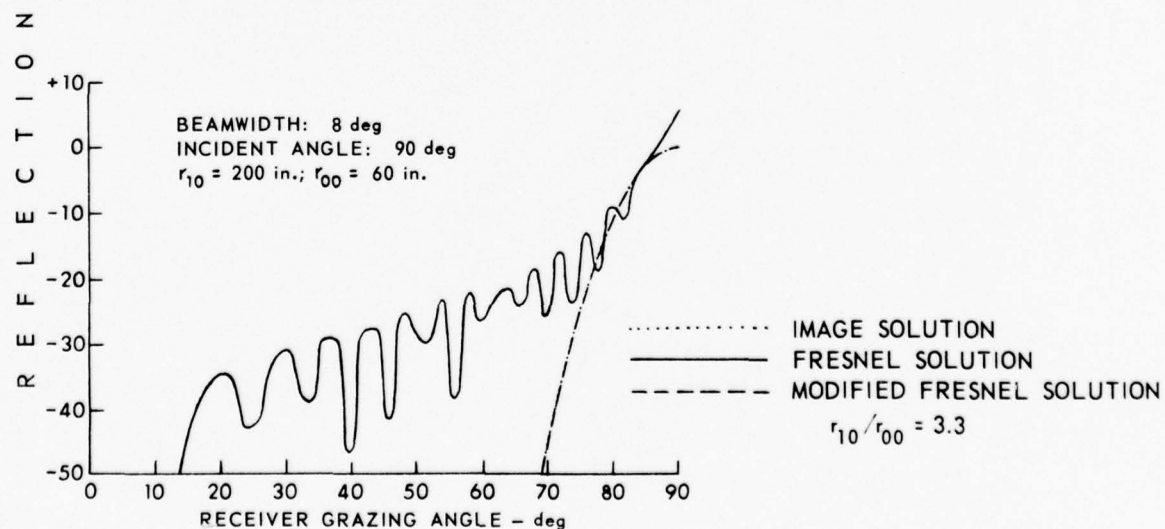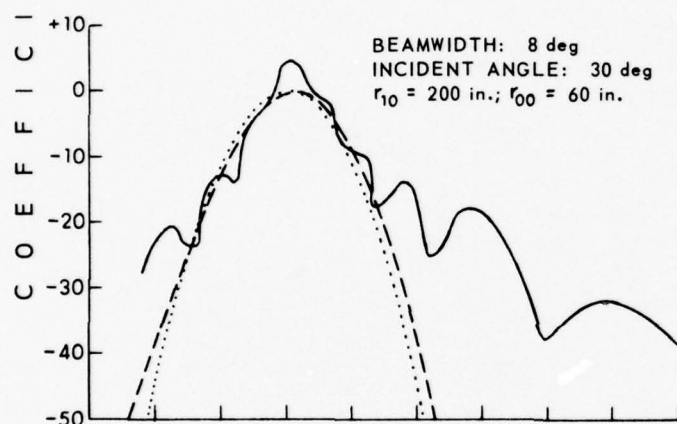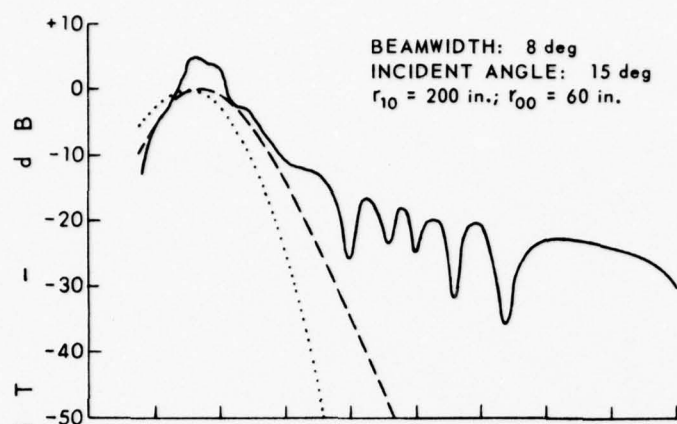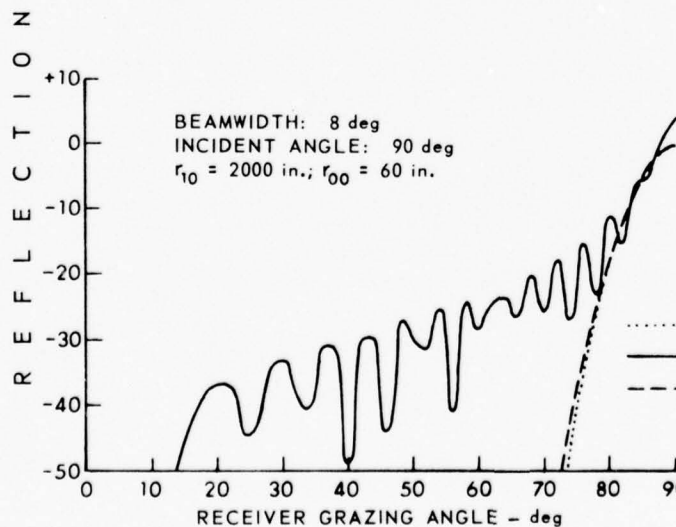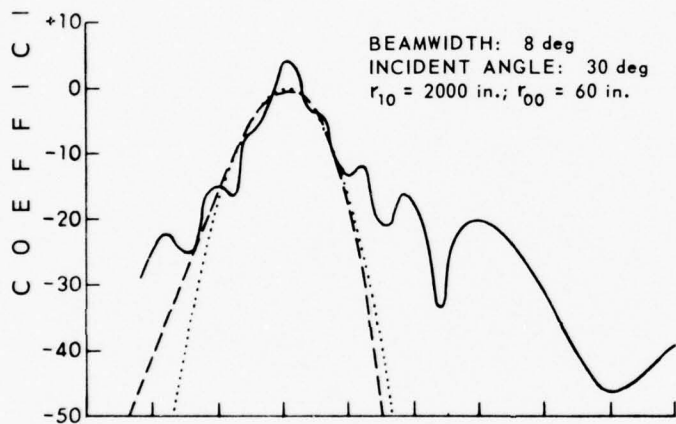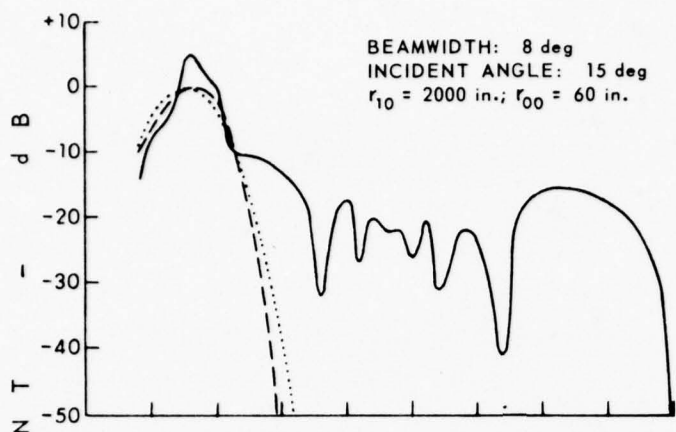
the Fresnel solution with an aperture type directivity pattern (Fresnel solution). It is interesting to note that at low grazing angles and for receiver ranges of the same magnitude as the source range, the distortion in the Modified Fresnel solution corresponds closely to the distortion in the directivity pattern (Eq. 23), but is reflected. As r_{lo}/r_{oo} increases, the distortion in the Modified Fresnel solution decreases until for large r_{lo}/r_{oo} the Modified Fresnel is nearly equal to the image solution.

II. CALCULATION OF REFLECTION COEFFICIENTS
FOR ROUGH ATTENUATING BOTTOMS

In this section the Rayleigh reflection coefficient is modified to account for bottom roughness. The method of Mackenzie[4] is followed so that the effects of roughness will be easily observable. From Dwg. AS-70-795 it is obvious that to include the effects of roughness it is only necessary to express the local incident and refracted angles, Φ_1 and Φ_2, in terms of the angles from the mean plane, θ_1 and θ_2, at each point on the surface. These are related through the angle β which, for a randomly rough surface, will be a stochastic variable. From the drawing it is clear that

$$\Phi_1 = \theta_1 + \beta \qquad , \qquad \Phi_2 = \theta_2 + \beta \qquad . \tag{30}$$

It is assumed that at any point on the surface, the reflection coefficient is given by the usual Rayleigh coefficient referenced to the tangent plane. Following Mackenzie,[4] this expression is

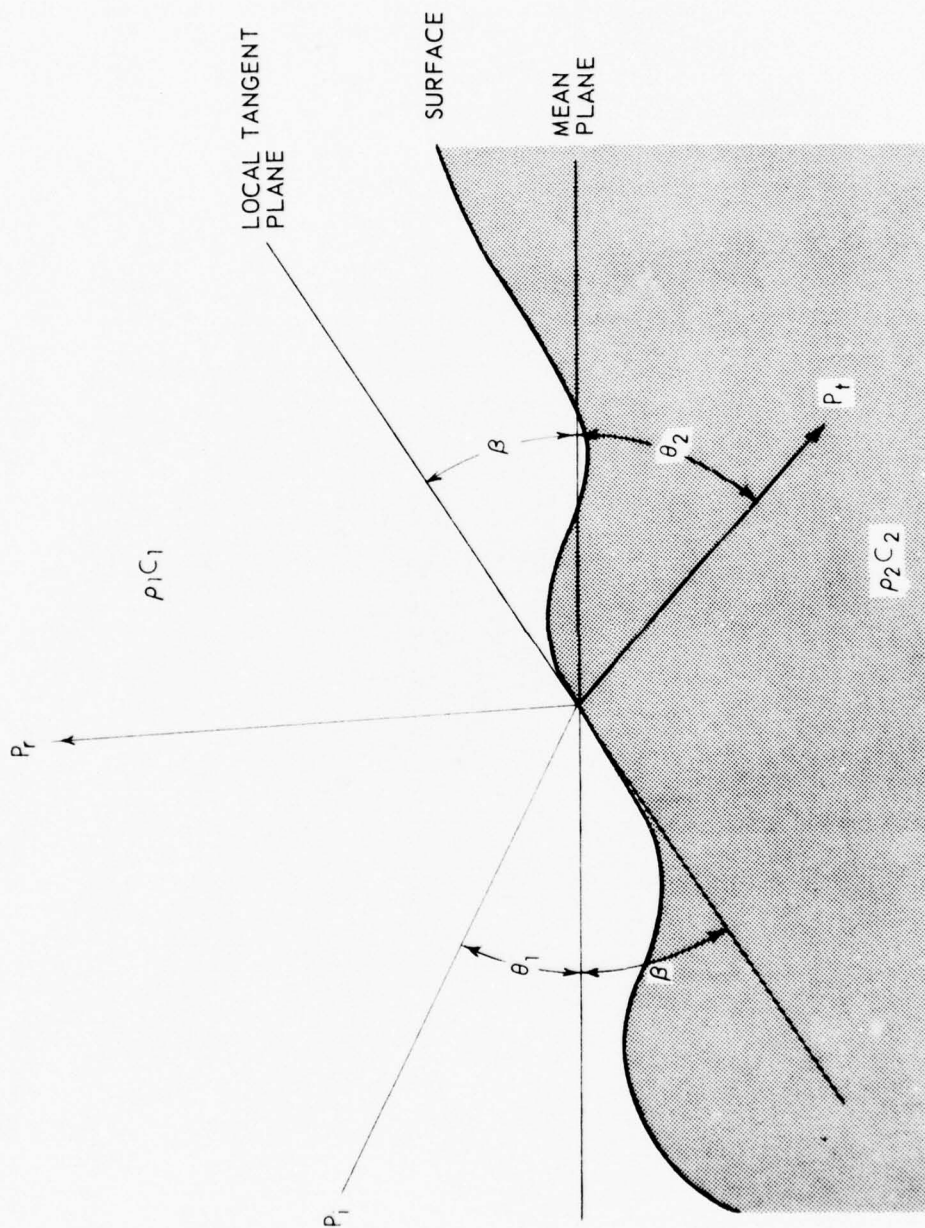$$R = \frac{p_r}{p_i} = \frac{1-S}{1+S} \qquad , \tag{31}$$

where

$$S = \frac{\rho_1 c_1 \sin\Phi_2}{\rho_2 c_2 \sin\Phi_1} \qquad , \tag{32}$$

so that

$$R = \frac{\rho_2 c_2 \sin\Phi_1 - \rho_1 c_1 \sin\Phi_2}{\rho_2 c_2 \sin\Phi_1 + \rho_1 c_1 \sin\Phi_2} \qquad . \tag{33}$$

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PENETRABLE ROUGH SURFACE GEOMETRY

Using Eq. (30) in Eq. (33) and the appropriate trigonometric identities gives

$$R = \frac{\rho_2 c_2(\sin\theta_1\cos\beta + \cos\theta_1\sin\beta) - \rho_1 c_1(\sin\theta_2\cos\beta + \cos\theta_2\sin\beta)}{\rho_2 c_2(\sin\theta_1\cos\beta + \cos\theta_1\sin\beta) + \rho_1 c_1(\sin\theta_2\cos\beta + \cos\theta_2\sin\beta)} \qquad . \quad (34)$$

Now let

$$\eta = \tan\beta \qquad , \qquad (35)$$

so that η is the slope at the given point on the surface, and let

$$A_- = \rho_2 c_2 \sin\theta_1 - \rho_1 c_1 \sin\theta_2 \qquad ,$$

$$A_+ = \rho_2 c_2 \sin\theta_1 + \rho_1 c_1 \sin\theta_2 \qquad ,$$

$$\qquad\qquad (36)$$

$$B_- = \rho_2 c_2 \cos\theta_1 - \rho_1 c_1 \cos\theta_2 \qquad , \text{ and}$$

$$B_+ = \rho_2 c_2 \cos\theta_1 + \rho_1 c_1 \cos\theta_2 \qquad .$$

This gives

$$R = \frac{A_- + \eta B_-}{A_+ + \eta B_+} = \frac{(A_- + \eta B_-)}{A_+}\left(1 + \frac{B_+}{A_+}\eta\right)^{-1} \qquad . \qquad (37)$$

Expanding this in terms of η gives

$$R = \frac{A_-}{A_+} + \left(\frac{A_- B_+ - A_+ B_-}{A_+^2}\right)\sum_{K=1}^{\infty}(-1)^K\left(\frac{B_+}{A_+}\right)^{K-1}\eta^K \qquad , \qquad (38)$$

where A_-/A_+ is the Rayleigh coefficient for a plane surface.

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The parameter S may now be redefined in terms of the angles θ_1 and θ_2 as

$$S' = \frac{\rho_1 c_1 \sin\theta_2}{\rho_2 c_2 \sin\theta_1} \quad . \tag{39}$$

After a little algebraic manipulation the Rayleigh coefficient becomes

$$R = \left\{ \left| \frac{1-S'}{1+S'} \right| \right\} + \left\{ \frac{2S'\left(\frac{\tan\theta_1}{\tan\theta_2} - 1 \right)}{\tan\theta_1 \left[1+S' \right]^2} \right\} \sum_{K=1}^{\infty} (-1)^K \eta^K \left\{ \tan\theta_1 \frac{\left[1 + \frac{\tan\theta_1}{\tan\theta_2} S' \right]}{\left[1+S' \right]} \right\}^{K-1} \quad . \tag{40}$$

Since η is a random variable, to find $\langle R \rangle$ it is only necessary to replace η by $\langle \eta \rangle$ in the series above. It is assumed that η has a zero mean Gaussian distribution. This implies that all odd moments $\langle \eta^{2K+1} \rangle$ are zero.

To include the effects of attenuation in the bottom it is now necessary to allow S' to be complex:

$$S' = x + iy \quad . \tag{41}$$

The quantities x and y are related to Mackenzie's parameters g and h by

$$x + iy = \frac{h+ig}{\sigma \sin\theta_1} \quad , \tag{42}$$

where σ is the impedance ratio at the boundary. If only the first nonzero term of the series is retained, $\langle R \rangle$ may be put in the form

$$\langle R \rangle = \frac{A'+iB'}{C'+iD'} \quad , \tag{43}$$

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where

$$A' = \left\{ \left[(1+x)^2 - y^2\right](1-x) + 2y^2(1+x) + 2\langle\eta^2\rangle \left(\frac{\tan\theta_1}{\tan\theta_2} - 1\right) \right.$$

$$\left. \cdot \left[x + \frac{\tan\theta_1}{\tan\theta_2}(x^2 - y^2)\right] \right\} \quad ,$$

$$B' = \left\{ -y\left[(1+x)^2 - y^2\right] + 2y^2(1-x^2) + 2\langle\eta^2\rangle \left(\frac{\tan\theta_1}{\tan\theta_2} - 1\right)\left[y + \frac{\tan\theta_1}{\tan\theta_2} 2xy\right] \right\} \quad .$$

$$C' = \left\{ \left[(1+x)^2 - y^2\right](1+x) - 2y^2(1+x) \right\} \quad , \text{ and} \tag{44}$$

$$D' = \left\{ y\left[(1+x)^2 - y^2\right] + 2y(1+x)^2 \right\} \quad .$$

Finally, the magnitude of the reflection coefficient including attenuation and roughness is given by

$$|\langle R\rangle|^2 = \left(\frac{1}{C'^2 + D'^2}\right)^2 \left[\left(A'C' + B'D'\right)^2 + \left(B'C' - A'D'\right)^2\right] \quad . \tag{45}$$

In calculating the RMS slope $\langle\eta^2\rangle$, it was assumed that the surface was shadowed at all points where the slope was less than the tangent of the incident angle. This assumption results in a simple change in the limits of the integral that defines $\langle\eta^2\rangle$ and gives a value of $\langle\eta^2\rangle$ that significantly differs from the true RMS slope only at small grazing angles.

Three separate fluid bottoms with different densities and velocities were considered. These correspond to three of the samples used by Mackenzie. The calculated quantity is the magnitude of the reflection coefficient as given by Eq. (45). This equation does not

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include the angular distribution of the scattered field, so it is not
an equation for the full scattering coefficient. It simply represents
the change in the reflection coefficient resulting from the introduction
of random roughness into the geometry of the problem. In subsequent
reports the angular distribution will be combined with these results to
give an equation for the scattering coefficient for penetrable rough
surfaces.

All of the calculated curves were evaluated at 5.0 kHz for ease of
comparison with Mackenzie's data. Several different values of RMS slope
were calculated, but only the one which produced the largest variations
in the reflection coefficient was plotted. Drawings AS-70-796 and
AS-70-797 show cases where $c_2/c_1 < 1$. For these cases it is possible to
have an angle of intromission; however, neither of the curves shown exhibit
this behavior because of the assumed attenuation. Drawing AS-70-798
shows a case where $c_2/c_1 > 1$. Again, the critical angle behavior of this
bottom is absent due to the attenuation. It should be noted that where
$c_2/c_1 < 1$, the reflection coefficient that includes roughness lies, for
the most part, below the plane reflection coefficient, while for
$c_2/c_1 > 1$, the opposite is true.

In conclusion, the overall effect of including roughness in the
reflection coefficient is very slight. In none of the cases considered
was the variation more than about 2 dB for the roughest bottoms and,
indeed in most cases, the variation was only a few tenths of a decibel.

CLAYEY FINE SILT – 5.0 kHz
MACKENZIE SAMPLE No. 27
ρ_1 = 1.0245 g/cc
C_1 = 4945.0 ft/sec
ρ_2 = 1.60 g/cc
C_2 = 4905.0 ft/sec
$\frac{\alpha}{\beta}$ = 0.0104
ATTENUATION = 1.42 dB ft AT 30 kHz

GRAZING ANGLE θ, – deg

REFLECTION COEFFICIENT

NO ROUGHNESS

rms SLOPE = 1.0
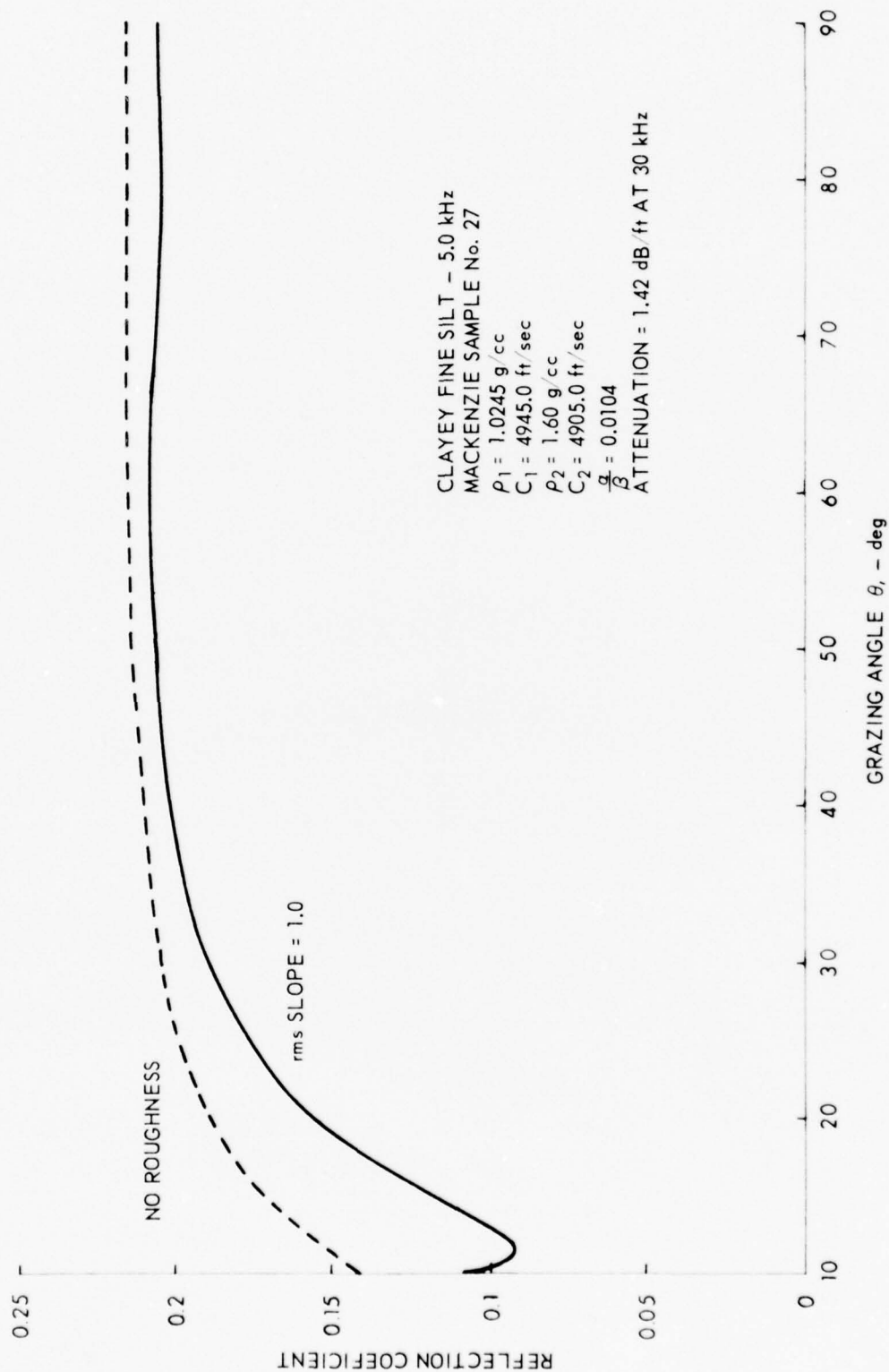
REFLECTION COEFFICIENT AS A FUNCTION OF GRAZING
ANGLE FOR A CLAYEY FINE SILT BOTTOM

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REFLECTION COEFFICIENT AS A FUNCTION OF GRAZING
ANGLE FOR A SAND – SILT – CLAY BOTTOM

SAND – SILT – CLAY – 5.0 kHz
MACKENZIE SAMPLE No. 13
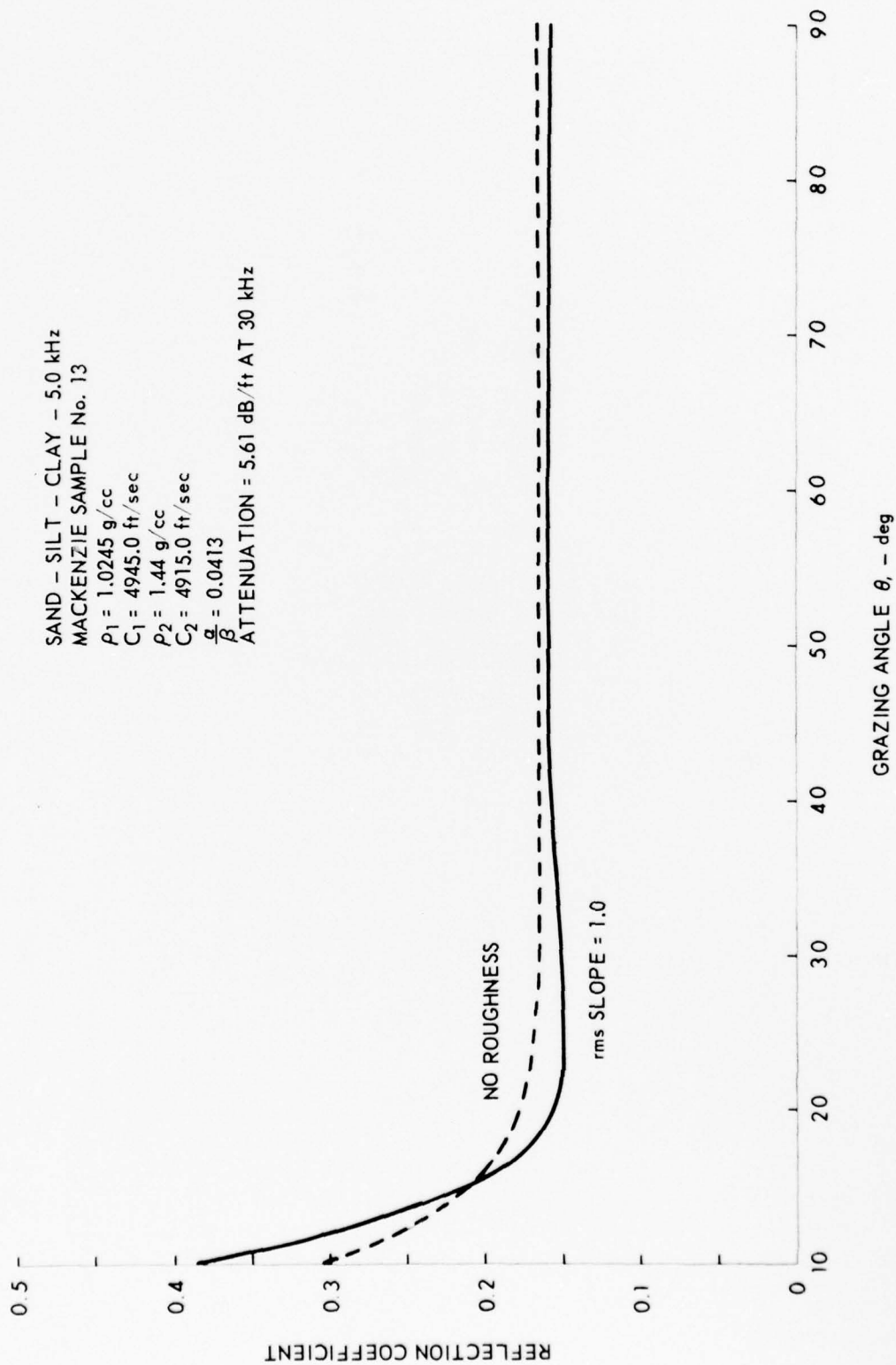
ρ_1 = 1.0245 g/cc
C_1 = 4945.0 ft/sec
ρ_2 = 1.44 g/cc
C_2 = 4915.0 ft/sec
$\frac{\alpha}{\beta}$ = 0.0413
ATTENUATION = 5.61 dB/ft AT 30 kHz

NO ROUGHNESS

rms SLOPE = 1.0

GRAZING ANGLE θ, – deg

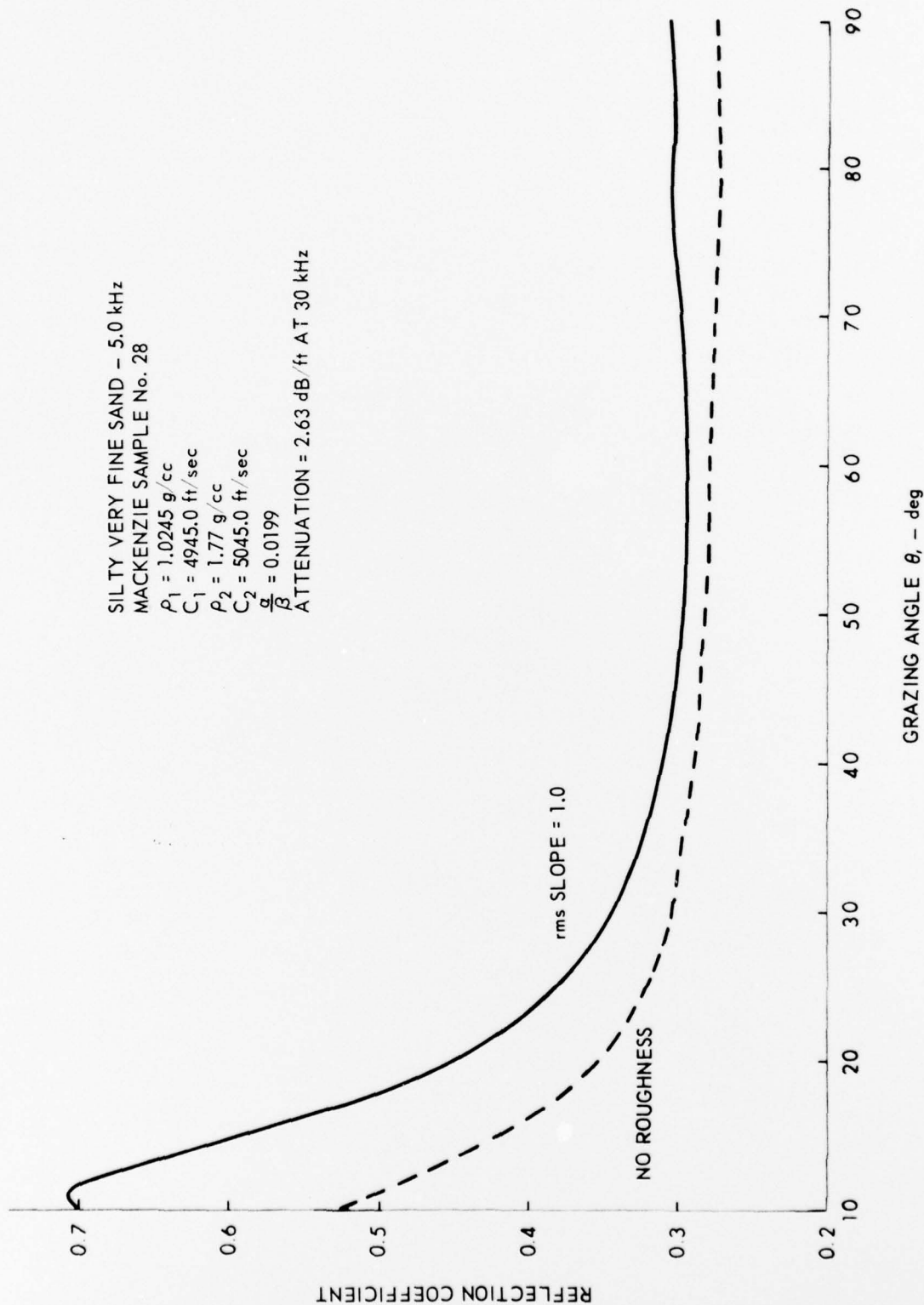REFLECTION COEFFICIENT

REFLECTION COEFFICIENT AS A FUNCTION OF GRAZING
ANGLE FOR A SILTY VERY FINE SAND BOTTOM

SILTY VERY FINE SAND – 5.0 kHz
MACKENZIE SAMPLE No. 28
P_1 = 1.0245 g/cc
C_1 = 4945.0 ft/sec
P_2 = 1.77 g/cc
C_2 = 5045.0 ft/sec
$\frac{\alpha}{\beta}$ = 0.0199
ATTENUATION = 2.63 dB/ft AT 30 kHz

rms SLOPE = 1.0

NO ROUGHNESS

GRAZING ANGLE θ, – deg

REFLECTION COEFFICIENT

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III. ACOUSTIC FIELD IN A SURFACE DUCT
WITH A ROUGH BOUNDARY

A. Introduction

The propagation of acoustic waves in a surface duct with an
irregular boundary is treated by means of Green's functions. Ray
theory has been applied to this problem by various authors,[5-10] but
for shallow water, low-frequency propagation ray theory is inadequate.
Normal mode wave theory has also been used[11-22] in examining this
problem. A useful survey of both ray theory and wave approaches has
recently been given by Schulkin.[23]

The following treatment will be very similar to that of Bucker's[22]
but will use Green's functions and a different impedance condition in
the basic integral. The impedance condition now depends on the nature
of the medium as well as the random boundary. A rough boundary can be
approximated by a flat boundary with some effective impedance (or
reflection coefficient) in the case of wave guide propagation, but
the expression for this impedance (or reflection coefficient) will be
completely different from the forms for reflection from a single
uneven surface.[14,17,19,20]

When the rough surface is replaced by an impedance condition
(or effective reflection coefficient), the propagation problem can be
solved by classical methods. As Bucker[22] indicates, this approach is
useful for the case where both source and receiver are in the duct.
This case is referred to as first-order scattering (scattering out of
a mode). When the source is in the duct and the receiver is below the

duct, then one must consider second-order scattering (scattering into a mode). The treatments of second-order scattering given by Schweitzer[7] and Van Ness[8] are believed to be in error. The present treatment will only consider the first-order effects of the rough boundary.

B. Green's Function Solution

The wave equation for a point monopole source of unit strength, with angular frequency ω, is given by

$$\Delta^2 p - \frac{1}{c^2(z)} \frac{\partial^2 p}{\partial t^2} = -4\pi \ \delta \ (\bar{r} - \bar{r}_o) e^{-i\omega t} \quad , \qquad (46)$$

where

1) p represents the pressure,

2) $c(z)$ is the sound velocity (variable in the coordinate z), and

3) $\delta(\bar{r} - \bar{r}_o)$ is the three dimensional Dirac delta function.

If cylindrical coordinates (r, θ, z) with azimuthal symmetry are assumed and the time factor of $\exp(-i\omega t)$ is suppressed, then

$$\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial G}{\partial r} \right) + \frac{\partial^2 G}{\partial z^2} + k^2(z) G = -\frac{2}{r} \delta(r) \ \delta(z - z_o) \quad , \qquad (47)$$

where 1) $k(z) = \omega/c(z)$ represents the wave number, and 2) the source is located at $z = z_o$ and $r = o$, as indicated in Dwg. AS-70-767. The vertical depth coordinate z varies from $o \leq z \leq \infty$, and the range coordinate r varies from $o < r < \infty$.
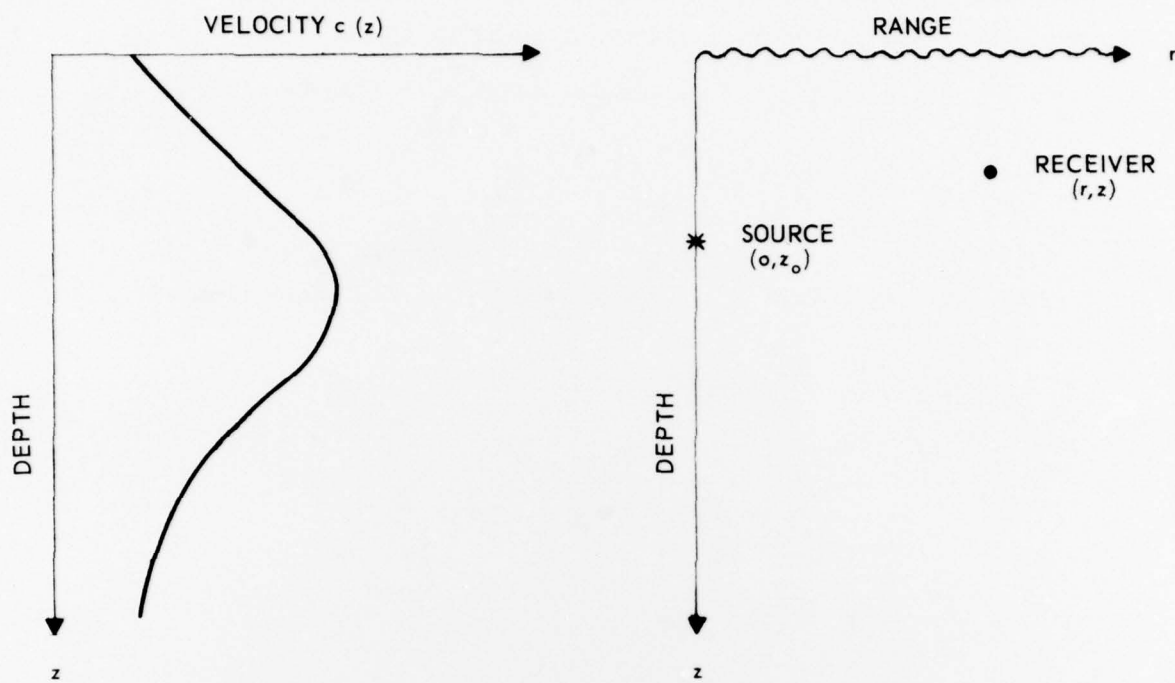
DIAGRAM OF WAVEGUIDE SHOWING LOCATION
OF SOURCE AND RECEIVER

The pressure is then $\exp(-i\omega t)$ times the Green's function G of Eq. (47). The boundary conditions imposed on Eq. (47) are

a) G must satisfy a radiation condition for $r \to \infty$ and $z \to \infty$.

b) $G \left/ \dfrac{\partial G}{\partial z} \right|_{z = 0} = \gamma$,   (48)

where γ represents the impedance (or corresponding effective reflection coefficient) condition found by the prescription given by Lysanov.[20]

In the Green's function approach, Eq. (47) is separated into the following forms:

$$\left[L_r + \lambda_1 r\right] G_1(r, \lambda_1) = -2\delta(r)   ,   (49)$$

$$\left[L_z - \lambda_1\right] G_2(z, z_0, -\lambda_1) = -\delta(z - z_0)   ,   (50)$$

where the differential operators are defined by

$$L_r = \frac{d}{dr}\left(r \frac{d}{dr}\right)   , \text{ and}   (51)$$

$$L_z = \frac{d^2}{dz^2} + k^2(z)   , \text{ and}   (52)$$

where λ_1 is the separation constant. G_1 and G_2 are defined as the resolvent Green's functions when they satisfy Eq. (49) and Eq. (50), respectively, and are subject to the previously stated boundary conditions. The separation constant is a complex parameter so restricted as to assure a unique solution for Eqs. (49-50).

The Green's function G of Eq. (47) is now given by a complex convolution of G_1 and G_2:

$$G(r,z,z_o) = \frac{1}{2\pi i} \int_C G_1(r,\lambda_1) \, G_2(z,z_o,-\lambda_1) \, d\lambda_1 \quad , \qquad (53)$$

where the contour C separates the singularities of G_1 and G_2 and can be closed around those singularities of either one which allow a uniqueness condition to be satisfied for G_1 and G_2.

The solutions to Eq. (49) and Eq. (50) that satisfy the required boundary conditions will be given now. For the $\exp(-i\omega t)$ time factor, these solutions are given by

$$G_1(r,\lambda_1) = i\pi H_o^1 (\xi r)(\xi = \lambda_1^{1/2}, o < \arg \lambda_1 < 2\pi) \quad , \qquad (54)$$

and

$$G_2(z,z_o,-\lambda_1) = \frac{n_1(z,\xi)\left[n_2(z_o,\xi) - X^- n_1(z_o,\xi)\right.}{W(n_2,n_1)} \; z_o < z < \infty \quad , \; (55)$$

$$G_2(z,z_o,-\lambda_1) = \frac{n_1(z_o,\xi)\left[n_2(z,\xi) - X^- n_1(z,\xi)\right]}{W(n_2,n_1)} \; o < z < z_o \quad , \; (56)$$

where $H_o^1(\xi r)$ is the Hankel function of the first kind, and

$$X^- = \frac{n_2(o,\xi) - \gamma n_2'(o,\xi)}{n_1(o,\xi) - \gamma n_1'(o,\xi)} \quad . \qquad (57)$$

The Wronskian of n_2 and n_1 is represented by $W(n_2,n_1)$. where n_1 is a solution to the homogeneous part of Eq. (50) which has outgoing waves

at $z \to +\infty$, and n_2 which has outgoing waves at $z \to -\infty$. The primes indicate differentiation with respect to z.

When Eq. (54) and Eq. (55) are used in Eq. (53), the Green's function for Eq. (47) is obtained:

$$G(r,z,z_o) = \int_C \frac{n_1(z_<,\xi)\left[n_2(z_>,\xi) - X^- n_1(z_>,\xi)\right] H_o^1(\xi r)\xi d\xi}{W(n_2,n_1)} \quad , \quad (58)$$

where $z_<$ and $z_>$ denote the smaller or larger, respectively, of the variables z and z_o.

When Eq. (58) is integrated by Cauchy's residue theorem, the normal modes (plus any branch line integrals) are obtained. The poles yielding the normal modes are given by the zeroes of the expression $\left[n_1(o,\xi) - \gamma n_1'(o,\xi)\right]$.

Equation (58) represents the desired formula for the propagation of sound in a surface channel with a rough boundary. It should be noted, however, that because of the formulation, only first-order effects of the rough surface are considered.

Future work will deal with the application of Eq. (58) to specific velocity-depth profiles, the first being the bilinear profile. The results will then be compared with Bucker's,[22] who also used the bilinear profile with a different interpretation for the impedance condition.

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DOCUMENT CONTROL DATA - R & D

Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY (Corporate author)	2a. REPORT SECURITY CLASSIFICATION
Applied Research Laboratories The University of Texas at Austin Austin, Texas 78712	UNCLASSIFIED, ▮ 2b. GROUP ---

3. REPORT TITLE

Quarterly Progress Report for April, May, and June 1970
Under Contract N00024-70-C-1279

4. DESCRIPTIVE NOTES (Type of report and inclusive dates)

Quarterly Progress Report for the period 1 April - 30 June 1970

5. AUTHOR(S) (First name, middle initial, last name)

6. REPORT DATE	7a. TOTAL NO. OF PAGES	7b. NO. OF REFS
21 July 1970	40	23

| 8a. CONTRACT OR GRANT NO.
N00024-70-C-1279
b. PROJECT NO.
SF 11552-001, Task 8118
c.
d. | 9a. ORIGINATOR'S REPORT NUMBER(S)

9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report)
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13. ABSTRACT

In Section I it is shown by using the simplest reflecting surface--an infinite, pressure release plane surface-- that at least the Fresnel phase approximation and a reasonable directivity pattern approximation are necessary before accurate predictions in surface reflection and scattering problems can be made. The average Rayleigh reflection coefficient for random penetrable surfaces of various roughnesses is calculated in Section II. In Section III an integral expression is derived for the acoustic field of a point source in a surface duct with a rough boundary. The integral was obtained by Green's functions and includes the rough surface condition by means of an effective impedance condition which also depends on the properties of the medium. (U-▮)

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	ROLE	WT	ROLE	WT	ROLE	WT
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Fresnel Phase Approximation						
Gaussian Directivity Pattern						
Aperture Directivity Pattern						
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